Chapter No -25 ( Circles )

HINTS/SOLUTIONS
EXCERCISE (1)

1. Let centre be \((h, k)\)
   \[
   \therefore \text{radius of circle} = CD = |k|
   \]
   In \(\triangle ABC\), \((AC)^2 = (AB)^2 + (BC)^2\)
   \[
   \Rightarrow k^2 = (2)^2 + (h)^2
   \]
   \[
   \Rightarrow k^2 - h^2 = 4
   \]
   \[
   \therefore \text{Locus of centre is} \quad y^2 - x^2 = 4
   \]
   \[
   \Rightarrow \text{Locus of centre is hyperbola}
   \]

2. Quadrilateral \(PQCR\) is cyclic
   \[
   \therefore \text{circumcentre of} \quad \triangle PQR = (0, 2)
   \]

3. \[
   \sqrt{3}y^2 - 4xy + \sqrt{3}x^2 = 0
   \]
   \[
   \Rightarrow (\sqrt{3}y - x)(y - \sqrt{3}x) = 0
   \]
   \[
   \therefore y = \frac{x}{\sqrt{3}} \quad \text{and} \quad y = \sqrt{3}x
   \]

4. \[
   \text{from figure,} \quad \angle AOB + \angle ACB = 180^\circ
   \]
\[ \therefore \angle ACB = 150^\circ \]

Apply cosine rule for \( \angle ACB \)

\[ \cos(150^\circ) = \frac{9 + 9 - (AB)^2}{2(3)(3)} \]

\[ \Rightarrow AB = 3 \left( \frac{\sqrt{3} + 1}{\sqrt{2}} \right) \text{ units.} \]

6.

Chord \( AB \) subtends double the angle at ‘O’ which it subtends at point \( P \) for circle \( x^2 + y^2 = 9 \).

In \( \triangle PDC \), \( \tan \theta = \frac{1}{2\sqrt{3}} \)

Now, in \( \triangle OBQ \), \( \cos 2\theta = \frac{OB}{OQ} \)

\[ \Rightarrow 1 - \frac{\tan^2 \theta}{1 + \tan^2 \theta} = \frac{3}{OQ} \Rightarrow OQ = \frac{27}{7} \]

\[ \therefore \text{Locus of point } Q \text{ can be given by:} \]

\[ x^2 + y^2 = \left( \frac{27}{7} \right)^2 \]

7. \( 16x^2 + 4y^2 = 64 \)

\[ \Rightarrow \frac{x^2}{4} + \frac{y^2}{16} = 1 \]

If common tangent is not porrible, then both curves neither touches nor intersect each other

\[ \therefore r < 2 \text{ or } r > 4 \]

\[ \Rightarrow r \in R - [2, 4] \]

8. \( y = mx + 2\sqrt{1 + m^2} \) is common tangent to \( x^2 + y^2 = 4 \) and \( (x - 2a)^2 + y^2 = 4 \)

9. For given circle \( C = (-1, 1) \) and radius is 10 units perpendicular distance \( CP = 8 \Rightarrow AP = 6 \)

\[ 3x - 4y = 33 \]

\[ y = mx + 2\sqrt{1 + m^2} \] pass through \((a, 0)\) and \( m < 0 \) because \( y \)-intercept is +ve

\[ \therefore 0 = ma + 2\sqrt{1 + m^2} \Rightarrow m = \frac{-2}{\sqrt{a^2 - 4}} \]

10. For given circle \( C = (-1, 1) \) and radius is 10 units perpendicular distance \( CP = 8 \Rightarrow AP = 6 \)

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\[ \therefore 0 = ma + 2\sqrt{1 + m^2} \Rightarrow m = \frac{-2}{\sqrt{a^2 - 4}} \]
If figure, \( \sin \theta = \frac{r}{2r} = \frac{1}{2} \) \( \Rightarrow \) \( 2\theta = 60^\circ \)

11. For family of lines \( ax + by + c = 0 \), if \( a + b + c = 0 \) then lines pass through point \((1, 1)\) given family of circles is \( x^2 + y^2 - 4x - 4y + \lambda = 0 \)
\[ \Rightarrow (x - 2)^2 + (y - 2)^2 + \lambda - 4 = 0 \]

from figure, \( PQ \) chord is of maximum length if it is diameter of the circle
\[ \Rightarrow (2 - 1) = m(2 - 1) \]
\[ \Rightarrow m = 1 \]
\[ \therefore \] required line is \( y - x = 0 \)

12. Let the smallest circle be \( x^2 + y^2 + 2gx + 2fy + c = 0 \)
\[ \therefore \] By condition of orthogonality
\[ 0 + 0 = c - 1 \Rightarrow c = 1 \]
\[ 2g(4) + 2f(4) = 1 - 33 \]
\[ \Rightarrow g + f = -4 \] \( \ldots \) (i)
radius \( = \sqrt{g^2 + f^2 - c} = \sqrt{g^2 + f^2 - 1} \)
\[ \Rightarrow \text{radius} = \sqrt{g^2 + (4 + g)^2 - 1} \] (from eq. (i))
\[ = \sqrt{2g^2 + 8g + 15} \]
\[ = \sqrt{2(g + 2)^2 + 7} \]
\[ \Rightarrow r_{\text{min}} = \sqrt{7} \text{ at } g = -2 \]
\[ \therefore \text{Centre is } (-g, -f) = (2, 2) \]

13. Tangent to curve \( xy = 1 \) at \((1, 1)\) is given by \( x + y = 2 \)
\[ \therefore \] Let the equation of required circle be
\[ (x - 1)^2 + (y - 1)^2 + \lambda(x + y - 2) = 0 \]
\[ \Rightarrow x^2 + y^2 + (\lambda - 2)x + (\lambda - 2)y + (2 - 2\lambda) = 0 \]
If circle touches coordinate axes, then distance of centre from origin is \( \sqrt{2} \) times the radius

14. Let sides \( AB \) and \( AD \) are along the \( x \)-axis and \( y \)-axis respectively.

![Diagram](image_url)

Equation of line \( BC \):
\[ \alpha y - 3x + 3\alpha = 0 \]
By condition of tangency:
\[ \left| \frac{3\alpha - 9 + 3\alpha}{\sqrt{9 + \alpha^2}} \right| = 3 \]
\[ \Rightarrow \alpha = 4 \]
\[ \therefore \text{Area} = \frac{1}{2}(4 + 12) \times 6 = 48 \text{ square units} \]

15. Radical axis for any two circles is perpendicular to the side of triangle (as shown in figure)

\[ \therefore R \text{ is orthocentre of } \triangle ABC \]
16. Point 'P' is radical centre of circles
∴ Solving $S_1 - S_2 = 0$, $S_2 - S_3 = 0$, $S_3 - S_1 = 0$
Simultaneously
⇒ $P$ is (1, 1)

17. Let chord $PQ$ subtend an angle of $90^\circ$ at the origin

If locus point is $M(h, k)$, then equation of $PQ$ is given by:

$$\frac{y - k}{x - h} = \frac{k}{h} \Rightarrow hx + ky = h^2 + k^2$$

Now, homogenize the circle with $PQ$

$$x^2 + y^2 - (4x + 6y)\left(\frac{hx + ky}{h^2 + k^2}\right) - 3\left(\frac{hx + ky}{h^2 + k^2}\right) = 0$$

If $\angle POQ = 90^\circ$, in the above equation,

(coef of $x^2$) + (coef of $y^2$) = 0
⇒ $2h^2 + 2k^2 - 4\lambda - 6k - 3 = 0$
∴ Locus of $M$ is $2x^2 + 2y^2 - 4x - 6y - 3 = 0$

18. Let the centre of moving circle be $c(h, k)$

from figure, $(h - 3)^2 + (k - 3)^2 = (h + 2)^2$
⇒ $y^2 - 10x - 6y + 14 = 0$

19. Let $C_1$ be $x^2 + y^2 + 2gx + 2fy + c = 0$
By condition of orthogonality with $x^2 + y^2 = 4$
$0 + 0 = c - 4$
⇒ $c = 4$
Centre $(-g, -f)$ lies on $2x - 2y + 9 = 0$
∴ $2g = 2f + 9$
∴ Circle $c_1$ is $x^2 + y^2 + (2f + 9)x + 2fy + 4 = 0$
⇒ $(x^2 + y^2 + 9x + 4) + 2f(x + y) = 0$
By $S + \lambda L = 0$, fixed points are the intersection points of $y + x = 0$ and $x^2 + y^2 + 9x + 4 = 0$

20. Apply $L_1 L_2 + \lambda xy = 0$
∴ $(2x - y + 1)(x - 2y + 3) + \lambda xy = 0$
⇒ $2x^2 + 6x + 2y^2 - 3y + x - 2y + 3 + xy(\lambda - 5) = 0$
⇒ $\lambda = 5$
∴ Circle is $2x^2 + 2y^2 - 7x - 5y + 3 = 0$
⇒ Centre $\left(\frac{-7}{4}, \frac{-5}{4}\right)$

21. Let circle be $S + \lambda L = 0$ smallest circle is that member of the family of circles for which the centre lies on the line $L = 0$

22. Let the point of intersection be $P(h, k)$

line $AB$ is chord of contact
∴ $AB$ is given by $xh + yk - 12 = 0 \quad \text{(i)}$
equation (i) can also be given by $S_1 - S_2 = 0$
∴ $xh + yk - 12 = 0$ and $5x - 3y - 10 = 0$ represent the same line
∴ $\frac{h}{5} = \frac{k}{-3} = \frac{12}{10}$
⇒ $h = 6$, $k = \frac{-18}{5}$
∴ $P$ is $\left(6, \frac{-18}{5}\right)$
23. \( P \quad A \quad C \quad B \) is Cyclic Quadrilateral
\[
\therefore \quad \text{circle passing through } P, \ A, \ C \text{ and } B \text{ is:}
\]
\[
(x-1)(x-3)+(y-8)(y-2)=0
\]
\[
\Rightarrow \quad x^2 + y^2 - 4x - 10y + 19 = 0
\]

24. \[
\text{from figure, } \cos \alpha = \frac{\sqrt{3}}{3} \text{ and } \sin \alpha = \frac{1}{3}
\]
Now, \( \tan \alpha = \frac{\sqrt{3}}{R} \)
\[
\Rightarrow \quad R = 8 \text{ units.}
\]

25. \[
\sin \left( \frac{\theta}{2} \right) = \frac{a-b}{AB} = \frac{a-b}{a+b}
\]
\[
\therefore \quad \theta = 2 \sin^{-1}\left( \frac{a-b}{a+b} \right)
\]

26. \[
\text{Tangent } T_1 \text{ is } x+y=0 \text{ and tangent } T_2 \text{ is } 3x - 4y = 0
\]
\[
\therefore \quad \text{Slope of } T_1 \text{ and } T_2 \text{ is } -1 \text{ and } \frac{3}{4} \text{ respectively}
\]
\[
\Rightarrow \quad S = \left\{ x \in \mathbb{R} : \frac{-1}{3} < x < \frac{3}{4} \right\}
\]

27. Let the variable line through origin be \( y = mx \)
\[
A(1, 0)
\]
\[
(x-1)^2 + (y-1)^2 = 1
\]
\[
\text{CD} = \left| \frac{1-m}{\sqrt{1+m^2}} \right|
\]
\[
\therefore \quad PQ = 2 \sqrt{\frac{(1-m)^2}{1+m^2}} = 2 \sqrt{\frac{2m}{1+m^2}}
\]
\[
BB' = \frac{1}{\sqrt{1+m^2}}
\]
\[
\therefore \quad \Delta PBQ = \frac{1}{2} \sqrt{\frac{8m}{1+m^2}} \left( \frac{1}{\sqrt{1+m^2}} \right)
\]

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\[ \Delta = \frac{1}{2} \sqrt{8m} \]

\[ \frac{d\Delta}{dm} = 0 \Rightarrow 4(1 - 3m^2) = 0 \]

\[ \therefore \Delta \text{ is maximum if } m = \frac{1}{\sqrt{3}} \]

\[ \Rightarrow \text{Slope of variable line } = R^+ - \left( \frac{1}{\sqrt{3}} \right) \]

28. Tangent is given by \( y = -\frac{4}{3}x + 12 \sqrt{5} \left[ 1 + \frac{16}{9} \right] \)

\[ y = \frac{4x}{3} + 4 \Rightarrow 3y + 4x = 12 \]

\[ \Rightarrow \frac{x}{4} + \frac{y}{3} = 1 \]

for \( \Delta OAB \), Incentre \((I) = (1,1)\), circumcentre is \( \left( 2, \frac{3}{2} \right) \)

29. Quadrilateral \( ACEB \) is cyclic in nature

\[ (DA + DE)^2 > \sqrt{(DA)(DE)} > \frac{2}{1} \]

Now \( \frac{1}{DA} + \frac{1}{DE} > \frac{4}{DA + DE} \)

\[ \Rightarrow \frac{1}{DA} + \frac{1}{DE} > \frac{4}{AE} \] (from (iii))

\[ \Rightarrow \frac{1}{DA} + \frac{1}{DE} > \frac{2}{\sqrt{(DA)(DE)}} \] (from (i))

Now apply \( AM > GM > HM \) for \( DB \) and \( DC \)

\[ \frac{1}{DB} + \frac{1}{DC} > \frac{2}{(DB + DC)} \Rightarrow \frac{1}{BD} + \frac{1}{CD} > \frac{4}{BC} \] (from (ii))

As \( \frac{DA + DE}{2} > \sqrt{(DA)(DE)} \) \ldots(p)

\[ \frac{DB + DC}{2} > \sqrt{(DB)(DC)} \] \ldots(q)

Add equation (p) and (q)

\[ \therefore \frac{(AE + BC)}{2} > \sqrt{(DA)(DE)} + \sqrt{(DB)(DC)} \] (from (i))

\[ (AE + BC) > 4\sqrt{(AD)(DE)} \]

29. Quadrilateral \( ACEB \) is cyclic in nature

\[ (BD)(DC) = (AD)(DE) \] \ldots(i)

and \( BD + DC = BC \) \ldots(ii)

\[ AD + DE = AE \] \ldots(iii)

As 'D' is not the circumcentre of \( \Delta ABC \), hence \( DB \), \( DC \), \( DA \) and \( DE \) are distinct +ve real number

Apply \( AM > GM > HM \) for \( DA \) and \( DE \)

In \( \Delta ABC \), \( \sin(\angle BAC) = \frac{1}{2} \)
\[
\therefore \text{In } \triangle AED, \quad \frac{1}{2} = \frac{R}{R+3} \Rightarrow R = 3
\]

Similarly for \( C_1 \), \( r = \frac{1}{3} \)

31.

\[ \text{Statement (1) is false and statement (2) is true.} \]

32.

\[ \therefore \text{In circle } C_2, \quad (YP)(YB) = (YZ)(YQ) \quad \ldots \text{(i)} \]

\[ \therefore \text{In circle } C_1, \quad (YP)(YA) = (YX)(YQ) \quad \ldots \text{(ii)} \]

\[ \text{from (i) and (ii), } \frac{YP}{YQ} = \frac{YZ}{YB} = \frac{YX}{YA} \]

\[ \Rightarrow (YZ)(YA) = (YX)(YB) \]

\[ \Rightarrow (YZ) = (YX) \quad \left\{ \because YA = YB \right\} \]

\[ \therefore \quad \text{Statement (1) and (2) are true.} \]

33. Let \( \alpha = (2m+1) \), \( \beta = (2n+1) \), where \( m, n \in I \)

Now, \( (2m+1)^2 + (2n+1)^2 = 2012 \)

\[ \Rightarrow 4(m^2 + n^2) + 4(m+n) + 2012 \]

\[ \Rightarrow 4(m^2 + m^2 + n^2 + n) + 2 = 2012 \]

\[ \Rightarrow 8k + 2 = 2012 \quad \forall \quad m, n \in I \]

\[ \Rightarrow 8k + 2 = 2012 \quad (k \in \omega) \]

\[ \therefore \quad \text{No value of } k \text{ is possible} \]

\[ \therefore \text{Statement (1) and (2) are true and the reasoning is correct.} \]

34.

\[ \frac{PA + PB}{2} \geq \sqrt{(PA)(PB)} \Rightarrow (PA + PB) \geq 2(PT) \]

\[ \therefore \quad (PA + PB) \geq 2\sqrt{(-3)^2 + (1)^2 - 1} \quad (\because PT = \sqrt{S_1}) \]

\[ \Rightarrow \quad PA + PB \geq 6 \]

Both statement (1) and (2) are true and the reasoning is correct.

35. Circle \( C_1 \) is \((x - 2)^2 + (y - 3)^2 = 1\) and point \('P'\) lies outside the circle \( C_1 \)

36. Statement (2) is false.

37. Statement (1) is true because \( PACB \) is cyclic quadrilateral.

Statement (2) is also true but not the reasoning for statement (1)
38. Let point $(1, 0)$ and $(-1, 0)$ be $P$ and $Q$ respectively.

\[
\begin{align*}
AP &= BP &= CP &= DP = 1 \\
AQ &= BQ &= CQ &= DQ = 3
\end{align*}
\]

\[\Rightarrow A, B, C, D \text{ lies on unique circle.}\]

Statement (1) is true.

Statement (2) is false because through three collinear points, unique circle is not possible.

39. Let point $(1, 0)$ and $(-1, 0)$ be $P$ and $Q$ respectively.

\[\begin{align*}
\frac{AP}{AQ} &= \frac{BP}{BQ} = \frac{CP}{CQ} = \frac{DP}{DQ} = \frac{1}{3}
\end{align*}\]

\[\Rightarrow A, B, C, D \text{ lies on unique circle.}\]

\[\begin{align*}
\text{Statement (1) is true.} \\
\text{Statement (2) is false because through three collinear points, unique circle is not possible.}
\end{align*}\]

40. Let locus point 'C' be $(h, k)$.

\[\begin{align*}
OC &= 1 + |R| \\
\Rightarrow h^2 + k^2 &= 1 + k^2 + 2k \\
\Rightarrow x^2 &= 1 + 2|y| \quad \text{(where $|x| \geq 1$)}
\end{align*}\]

\[\text{Statement (1) is false.}\]
1. Equation of line \( CD \) in parametric form is
\[
\left\{ \begin{array}{l}
x = \frac{3 - \sqrt{3}}{2} \\
y = \frac{3 - \sqrt{3}}{2}
\end{array} \right. \quad \text{for centre } C, \quad r = \pm 1
\]
Two possible co-ordinates of centre are \((2\sqrt{3}, 2)\) and \((\sqrt{3}, 1)\)
According to the question \((\sqrt{3}, 1)\) lies on the same side where origin lies with respect to line \( PQ \).
\[
\Rightarrow \quad \text{Centre } C \text{ must be } (\sqrt{3}, 1)
\]
\[
\Rightarrow \quad \text{Equation of the circle is } (x - \sqrt{3})^2 + (y - 1)^2 = 1
\]
2. By simple geometry \( PD = \sqrt{3}(\triangle PQR \text{ is equilateral}) \)
Considering equation of \( PQ \) in parametric form co-ordinates of \( P \) and \( Q \) are \((2\sqrt{3}, 0)\) and \((\sqrt{3}, 3)\).
Point \( C \) divides the join of \( P \) and \( E \) in the ration \( 2 : 1 \)
Similarly, \( C \) divides join of \( Q \) and \( F \) in the ratio \( 2 : 1 \)
\[
\Rightarrow \quad \text{Co-ordinates of } E \text{ and } F \text{ are } \left( \frac{\sqrt{3}}{2}, \frac{3}{2} \right)
\]
and \((\sqrt{3}, 0)\)
3. Equation of line \( PR \) which is parallel to \( DE \) and passes through \( F \) is \((y - 0) = 0(x - \sqrt{3}) \Rightarrow y = 0.\)
Similarly, equation of line \( QR \) which is parallel to \( DF \) and passes through the point \( E \) is
\[
\left\{ \begin{array}{l}
y = \frac{3 - \sqrt{3}}{2} \\
x = \frac{3 - \sqrt{3}}{2}
\end{array} \right. \quad \Rightarrow \quad y = \sqrt{3}x.
\]
4. Let \( 'P' \) be \((\alpha, \sin \alpha)\)
As shown in figure, quadrilateral \( PACB \) is cyclic
\[
\Rightarrow \quad h = \left( \frac{\alpha - 3}{2} \right) \quad \text{and} \quad k = \frac{4 + \sin \alpha}{2}
\]
\[
\Rightarrow \quad 2k = 4 + \sin(2\alpha + 3) \Rightarrow \text{Locus of circumcentre is } 2y = 4 + \sin(2\alpha + 3)
\]
As per the given question,
\[
f(x) = 2 + \frac{1}{2}\sin(2\alpha + 3) \quad \cdots (1)
\]
As \(-1 \leq \sin \theta \leq 1 \quad \forall \ \theta \in \mathbb{R}\)
\[
\Rightarrow \quad \frac{3}{2} \leq f(x) \leq \frac{5}{2} \Rightarrow \left[ f(x) \right] = 1 \text{ or } 2
\]
Now \( S = \{ y : y = \left[ f(x) \right], x \in \mathbb{R} \} \)
\[
\Rightarrow \quad S = \{ 1, 2 \} \quad \Rightarrow \quad n(S) = 2
\]
5. \( g(x) = \lambda^2 \left[ \frac{1}{2} \sin(2\alpha + 3) + (6\alpha - 8) \right] \frac{1}{2} \sin \left( \frac{\pi}{2} + 2\alpha + 3 \right) \)
\[
\Rightarrow \quad g(x) = \frac{1}{2} \left[ \lambda^2 \left| \sin(2\alpha + 3) + (6\alpha - 8) \right| \right] \cos(2\alpha + 3)
\]
\[
\Rightarrow \quad \text{Fundamental period of } \left| \sin 2\alpha \right| + \cos 2\alpha \left| \frac{\pi}{4} \right|
\]
\[
\lambda^2 = 6\lambda - 8 \Rightarrow \lambda^2 - 6\lambda + 8 = 0
\]
\[
\Rightarrow \quad \lambda = 2 \text{ or } 4
\]
6. \(2 + \frac{1}{2}\sin(2x + 3) = e^{-|x|}\)

\[0 < e^{-|x|} \leq 1 \quad \text{and} \quad \frac{3}{2} \leq f(x) \leq \frac{5}{2}\]

\[\therefore \text{No solutions exists for} \quad f(x) = e^{-|x|}.

7.

Let \(\angle TOQ = \angle QOA = \alpha\)

\[\therefore \angle P = (90 - 2\alpha)\]

and \(PT = \tan 2\alpha\) from \(\triangle OAP\)

\[\tan 2\alpha = \frac{AP}{d} \quad \quad \text{(i)}\]

and from \(\triangle OTQ\)

\[\tan \alpha = \frac{1}{d}\]

Now \(OT + PT + AP + OA = 8\) (given)

\[\therefore AP = (8 - 2d - \tan 2\alpha)\]

From (1)

\[\tan 2\alpha = \frac{8 - 2d - \tan 2\alpha}{d} \quad \Rightarrow (d + 1)\tan 2\alpha = 8 - 2d\]

\[\therefore \frac{2\tan \alpha}{1 - \tan^2 \alpha} = \left(\frac{8 - 2d}{d + 1}\right)\]

\[\frac{\frac{1}{d}}{1 - \frac{1}{d^2}} = \frac{8 - 2d}{d + 1} \Rightarrow 2d^2 - 6d^2 + 8 = 0\]

\[d = 2\]

and hence \(\tan \alpha = \frac{1}{2}\)

Now \(QP\sin(90 - 2\alpha) = 1\)

\[\therefore QP = \frac{1}{\cos 2\alpha} = \frac{1(1 + \tan^2 \alpha)}{1 - \tan^2 \alpha} = \frac{1 + \frac{1}{4}}{1 - \frac{1}{4}}\]

\[QP = \frac{5}{3}\]

8. Circle 'C' is \((x+1)^2 + (y-2)^2 = 1\)

9.

Eq. of \(MN\) is given by \(S_1 - S_2 = 0\)

\[\therefore MN \text{ is } x + 1 = 0\]

\[\therefore MN \text{ is diameter of circle 'C'.}\]

10.

Let circle passing through \(A\) and \(B\) with centre at \(P\) be \(S = 0\)

\[\therefore (x-3)^2 + (y-4)^2 = (8)^2 \text{ is circle } S = 0\]

\[\Rightarrow x^2 + y^2 - 6x - 8y - 39 = 0\]

Now, family of circles passing through \(A\) and \(B\) can be given by \(S + \lambda \left(x^2 + y^2 - 25\right) = 0\)

\[\therefore (1 + \lambda)x^2 + (1 + \lambda)y^2 - 6x - 8y - 39 - 25\lambda = 0\]

Equation (i) satisfy the point \((-4, -4)\)

\[\therefore (1 + \lambda)(16) + (1 + \lambda)(16) + 24 + 32 - 39 - 25\lambda = 0\]

\[\Rightarrow \lambda = 7\]

Hence the required circle is:

\[3x^2 + 3y^2 + 3x + 4y - 18 = 0\]

11. If member of \(C_F\) is having minimum area than its diameter is \(AB\)

\[\therefore \text{Equation of } AB \text{ is } S_1 - S_2 = 0\]

\[\Rightarrow 6x + 8y + 14 = 0\]

\[\Rightarrow 3x + 4y + 7 = 0\]
If mid-point of \(AB\) is \(M\), then the radius of required circle is \(BM\).

\[
BM = \sqrt{25 - \frac{49}{25}} = \frac{24}{5} \text{ units.}
\]

12. Let point \(Q\) be \((\alpha, \beta)\).

\[
\therefore AB \text{ is the chord of contact for circle } S = 0 \text{ with respect to point } Q.
\]

\[
\Rightarrow \text{ Equations of } AB \text{ is } T = 0
\]

\[
\therefore ax + \beta y - 3(x + \alpha) - 4(y + \beta) - 39 = 0
\]

Now, comparing \(T = 0\) (i.e. equation of \(AB\)) with line \(3x + 4y + 7 = 0\).

\[
\Rightarrow \alpha = -3 \text{ and } \beta = -4
\]

13. Line passing through \(A\) and \(B\) is given by \(\frac{y - 7}{x - 3} = \frac{2}{-3}\).

\[
\Rightarrow 2x + 3y = 27
\]

\[
\therefore \text{ family of circle } C_F \text{ is } S + \lambda = 0
\]

\[
\Rightarrow (x - 6)(x - 3) + (y - 5)(y - 7) + \lambda (2x + 3y - 27) = 0
\]

\[
\Rightarrow x^2 + y^2 - 9x - 12y + 53 + \lambda (2x + 3y - 27) = 0
\]

Equation of common chord is given by \(S_1 - S_2 = 0\)

\[
\Rightarrow 5x + 6y - 56 - \lambda (2x + 3y - 27) = 0
\]

\[
\Rightarrow (5x + 6y - 56) + \lambda (27 - 2x - 3y) = 0
\]

\[
\therefore L_4 + \lambda L_2 = 0 \Rightarrow \text{ fixed point } (\alpha, \beta) \text{ is the point of intersection of } L_1 \text{ and } L_2
\]

\[
\Rightarrow \alpha = 2 \text{ and } \beta = \frac{23}{3} \Rightarrow \sqrt{\alpha + 3\beta} = 5
\]

14. If member of \(C_F\) is having minimum area than its diameter is \(AB\)
⇒ $x^2 + y^2 + (2\lambda - 2)x + (3\lambda + 2)y + \lambda + 2 = 0$ ... (i)
Now circle 'C' is orthogonal with the circle
$x(x + 2) + (y + 1)(y - 3) = 0.$
⇒ $x^2 + y^2 + 2x - 2y - 3 = 0$ ...
(ii)
Circle (i) and (ii) are orthogonal to each other
$\therefore (2\lambda - 2) + (3\lambda + 2)(-1) = \lambda + 2 - 3 \Rightarrow \lambda = -\frac{3}{2}$
Now circle 'C' is $x^2 + y^2 - 5x - \frac{5}{2}y + 1 = 0$
$\therefore$ Length of x-intercept (l)
$= 2\sqrt{\frac{25}{4} - \frac{1}{2}} \approx 4.8 \Rightarrow [l] = 4$
17. $x^2 + y^2 - 6x - 4y + 11 = 0$
⇒ $(x-3)^2 + (y-2)^2 = \left(\sqrt{2}\right)^2$
⇒ side of square $ABCD$ is 1 unit
\begin{align*}
\alpha^2 + \beta^2 + \gamma^2 + \delta^2 &= \left(s - \frac{1}{2}\right)^2 + \frac{1}{4} + \\
&\quad \left(t - \frac{1}{2}\right)^2 + \frac{1}{4} + \left(u - \frac{1}{2}\right)^2 + \frac{1}{4} + \left(v - \frac{1}{2}\right)^2 + \frac{1}{4} \\
\Rightarrow (\alpha^2 + \beta^2 + \gamma^2 + \delta^2) &\geq 4\left(\frac{1}{4}\right) \\
\therefore$ minimum value $= 1$
18.
\begin{align*}
\text{Centre of } C_1 \text{ and } C_2 &\text{ is } \left(\frac{5}{2}, \frac{3}{2}\right) \\
\therefore C_1: \left(x - \frac{5}{2}\right)^2 + \left(y - \frac{3}{2}\right)^2 = \frac{1}{2} \\
C_2: \left(x - \frac{5}{2}\right)^2 + \left(y - \frac{3}{2}\right)^2 = \frac{1}{4}
\end{align*}
19. (a) Eq. of tangent at $P(2, 2)$ is $(x + y = 4)$.
\therefore family of circles is $(x - 2)^2 + (y - 2)^2 + \lambda(x + y - 4) = 0.$
$x + y = 4$ is also tangent to
$x^2 + y^2 + 2x + 2y - 16 = 0$ at $P(2, 2)$
\therefore $(x - 2)^2 + (y - 2)^2 + \lambda(1 + 1)^2 + (1 + 1)^2 - 18 = 0$
is also parable.
(b) $\rightarrow q, r$
(c) $\rightarrow q, r$
(d) $\rightarrow s, q$
20. Let the fixed circle be $x^2 + y^2 + 2gx + 2fy + c = 0$.
\therefore By condition of tangency
\[\left|\frac{a(-g) + b(-f) + 1}{\sqrt{a^2 + b^2}}\right| = \sqrt{g^2 + f^2 - c}\]
⇒ $(ag + bf - 1)^2 = (a^2 + b^2)(g^2 + f^2 - c)$
⇒ $a^2(f^2 - c) + b^2(g^2 - c) + 2ag + 2bf - 2fgab - 1 = 0$
Now comparing above relation with the given condition
$12a^2 - 4b^2 + 8a + 1 = 0$
⇒ $f^2 - c = -12, g^2 - c = 4, 2g = -8, f = 0, fg = 0$
⇒ $c = 12, g = -4, f = 0$
\therefore Circle 'C' is $x^2 + y^2 - 8x + 12 = 0$
⇒ $(x - 4)^2 + y^2 = 4$
(a) $2(-4) + 0 = 12 - k \Rightarrow k = 20$
(b) Equation of chord $PQ$ is given by $S_1 - S_2 = 0$

$\therefore$ Equation of $PQ$ is $x = 3$

$\therefore PQ = 2\sqrt{3}$ or $\sqrt{12}$

(c) $OA = PB = \sqrt{12}$

\[ \tan \theta = \frac{2}{\sqrt{12}} = \frac{1}{\sqrt{3}} \Rightarrow \theta = 30^\circ \]

$\therefore \Delta OAB$ is equilateral triangle

$\Rightarrow r = \frac{1}{3} (\sqrt{12} \cos 30) = 1$ unit

$\therefore (20') = 20$

(d)

$0 \leq MN \leq 4$

$\therefore MN$ can be $\sqrt{12}, 3, \sqrt{10}$